A Toy Model of Renormalization and Reformulation

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I consider a special mechanical problem where the “particle acceleration” creates sound waves. Theoretical description should provide the total energy conservation. To introduce small “radiative losses” into the phenomenological mechanical equation, I advance a “self-interaction Lagrangian”, like in CED/QED. The “better coupled” mechanical and wave equations manifest unexpectedly wrong dynamics due to changes of their coefficients (masses). I show that these changes occur due to the mathematically erroneous self-interaction ansatz. I also show that renormalization of the fundamental constants in these new equations works: the exactly renormalized equations are in agreement with experiments and are correct, but they reveal a deeper physics - that of permanently coupled constituents of the “particle”. The latter was not contained in the self-interaction idea. Hence, with advancing a self-action Lagrangian we make mathematical and physical errors, renormalization is discarding harmful corrections fortunately compensating these errors, and the renormalized equations may sometimes accidentally coincide with the correct ones. The correct ones can be obtained directly if one proceeds from the right physical ideas about the observed phenomenon.

§1. Introduction

In this article I would like to sketch out how and when we may fall in conceptual and mathematical error while advancing our theories. It explains the true reason of difficulties encountered in course of coupling equations and the meaning of renormalizations of the fundamental constants. Briefly speaking, it happens because our understanding of physics and our way of coupling equations are wrong. At the same time, we may be very close to a better understanding and we may keep practically the same old equations in the correct formulation.

The difficulties encountered in CED and QED equations can be modeled quite reasonably with Classical Mechanics differential equations. It is instructive to analyze them to the end. Consideration in this article is carried out on purpose on a simple and feasible mechanical problem in order to demonstrate unambiguously that the fundamental constant modifications are not quantum, or relativistic, or nonlinear physical effects occurring with "bare" particles, but errors of our modeling. Classical electrodynamics (CED) and quantum electrodynamics (QED) and their historical developments will be used as examples to follow in our mechanical model.

In Chapter 2 I outline the experimental setup and the phenomenological equations establishable for this case. These equations are analogous to the CED equations without radiation reaction force. In Chapter 3 I advance an “interaction Lagrangian” in order to derive the radiation reaction force necessary for the energy conservation law. I show that despite achieving formally the “energy conservation law”, the new coupled equations differ from the original ones not only with the presence of a radiation reaction force, but also with other (kinetic) terms that essentially modify

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the dynamics of our variables. In Chapter 4 I fulfill renormalization of the modified constants and arrive at good equations resembling the old ones and containing the radiation reaction force, as was planned in the beginning of Chapter 3. I analyze the physics contained in these exactly renormalized equations and show that it finally corresponds to our special mechanical problem. Thanks to a specially designed (mechanical one-mode) problem, the origin of coefficient modifications is clearly visible whereas in QED it is obscured due to the perturbative (rather than exact) treatment of the "interaction Lagrangian".

§2. Phenomenon to describe

Let us consider a macroscopic probe body constructed with purpose to model a one-mode compound system (Fig. 1). We will suppose that it is a rather rigid shell of a diameter D with a core inside connected with springs modeling a 3D oscillator.

From the exterior the probe body looks as a solid ball and experimentalists do not know its true composition. The energy of oscillations is assumed to be small compared to the body kinetic energy, and experimentally one first establishes Newton equation for this body motion (hereafter also called a "particle"):

$$M_p \ddot{\mathbf{r}}_p = \mathbf{F}_{ext}(\mathbf{r}_p|t). \tag{2·1}$$

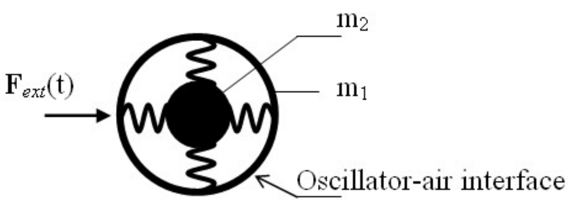

Fig. 1. Mechanical model of one-mode system.

Eq. (2·1) contains such fundamental physical notions as a particle position $\mathbf{r}_p$, a particle mass M_p, and an external force $\mathbf{F}_{ext}$, all of them being measurable physical quantities and nothing is bare. As our particle is not really point-like, Eq. (2·1) is written, strictly speaking, for its geometric center position. Eq. (2·1) is our analogue to the Lorentz equation for a point-like charge in an external electromagnetic field without radiation reaction force.

Now let us suppose that after our getting well accustomed to describing our "particle" with (2·1), experimentalists discover that *acceleration* of our probe body by an external force, creates some weak sound waves of a certain frequency ω. We will suppose that after the waves were discovered, some phenomenological description was progressively established. For example, experimentalists varied the force absolute value F_{ext} and its duration T and found an empiric equation for the oscillation amplitude A_{sound} (Fig. 2).

Finally, they found that during any force action the observed sound amplitude at some point of observation is a solution to the following driven oscillator equations

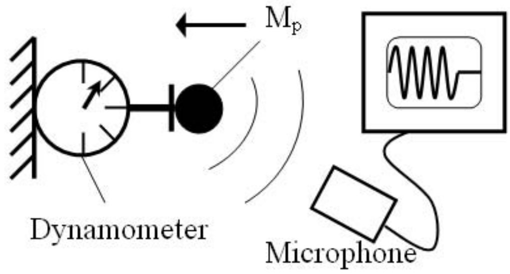

Fig. 2. Observing sound: collision of the body with a dynamometer.

(sound damping neglected for instance):

$$\ddot{A}_{sound} + \omega^2 A_{sound} = \alpha_{sound}(S)\ \mathbf{n}\cdot\ddot{\mathbf{r}}_p(t). \tag{2·2}$$

This equation is our analogue to the CED equation for the amplitude of an electromagnetic wave "sourced" with an electron acceleration due to an external force*). Here $\alpha_{sound}(S)$ is an experimentally measurable dimensionless coefficient of efficiency of wave excitation with the particle acceleration (a "pumping efficiency" or a "coupling constant") and the unit vector $\mathbf{n}$ points out the sound propagation direction. The body velocity $\dot{\mathbf{r}}_p$ (Doppler effect) and the distance S (sound retardation) are assumed reasonably small and thus inessential in our model.

The sound amplitude A_{sound} at a long distance $S \gg D$ from our "particle" depends, of course, in a known way on this distance and on the angle θ between the force and the propagation direction. We will assume that the measured sound amplitude is proportional to the true oscillator amplitude $\mathbf{r}_{osc}$, although we do not know much about the true oscillator yet due to, say, imperfections of experimental facility (for example, it is large compared to D) and difficulties in precise local measurements. In the following we will suppose that Eq. (2·2) can be rewritten factually for the 3D oscillator amplitude $\mathbf{r}_{osc}$ defined from the proportionality $A_{sound} \propto \mathbf{n}\cdot\mathbf{r}_{osc}$ (kind of limit $S \to D/2$). Then the oscillator Eq. (2·2) can be equivalently written via the oscillator (unknown) mass M_{osc} and (unknown) spring constant k, i.e., in a more canonical way:

$$M_{osc}\ddot{\mathbf{r}}_{osc} + k\,\mathbf{r}_{osc} = \alpha M_{osc}\ddot{\mathbf{r}}_p(t), \qquad \omega_{exp} = \sqrt{k/M_{osc}}. \tag{2·3}$$

The coupling constant α of the true oscillator does not depend on any distance by definition (as if it were something like $\alpha \approx \alpha_{sound}(S = D/2)$). The oscillator Lagrangian corresponding to (2·3) is then the following:

$$L_{osc} = \frac{M_{osc}\dot{\mathbf{r}}_{osc}^2}{2} - k\frac{\mathbf{r}_{osc}^2}{2} - \alpha M_{osc}\dot{\mathbf{r}}_{osc}\cdot\dot{\mathbf{r}}_p(t), \tag{2·4}$$

*) Indeed, the vector-potential in the Coulomb gauge $\mathbf{A}$ can be decomposed into Fourier series with amplitudes $\mathbf{q}_{\mathbf{k},\lambda}(t)$ obeying driven oscillator equations like $\ddot{\mathbf{q}}_{\mathbf{k},\lambda} + \omega^2_{\mathbf{k},\lambda}\mathbf{q}_{\mathbf{k},\lambda} \propto \mathbf{e}_{\mathbf{k},\lambda}\cdot\dot{\mathbf{r}}_e(t)$. The partial time derivative $\partial\mathbf{A}/\partial t$ determines the electric field "sourced" mainly with acceleration $\ddot{\mathbf{r}}_e(t)$.

where $\dot{\mathbf{r}}_p(t)$ is a known (given) function of time, namely, the solution to (2·1). The right-hand side term $\alpha M_{osc}\ddot{\mathbf{r}}_p(t)$ in (2·3) is then obtained from $\frac{d}{dt}\left(\partial L_{osc}/\partial\dot{\mathbf{r}}_{osc}\right)$ of the Lagrange equation. The oscillator mass M_{osc} can be estimated from the sound energy measurements with assuming that the whole initial oscillator energy is transmitted gradually to the air. As the measured body rigidity, mass M_p, and the size D do not provide the observed and unique frequency, our experimentalists continue performing and perfecting their experiments, and we theorists get busy with the existing description of this phenomenon (2·1), (2·3).

Remark: At this stage the known function of time $f(t) = \ddot{\mathbf{r}}_p(t)$ in (2·3) can be equivalently expressed via the external force value on the trajectory $\mathbf{F}_{ext}\left(\mathbf{r}_p(t)\right)/M_p$ from (2·1). The latter unambiguously shows that the point of external force application permanently belongs to the oscillator, but we will not base our theoretical development on this physical idea. Instead, we will continue expressing the pumping term via the particle *dynamical variable* $\ddot{\mathbf{r}}_p$, like a source in CED/QED. Note, as soon as the external force stops acting, the "wave system" (2·3) decouples from the "mechanical" one (2·1) and becomes "free". It makes an impression that the "mechanical" and the "wave" systems only interact during acceleration phase, otherwise they are "independent".

So we may well have two equations established experimentally with help of macroscopic measuring devices describing some non trivial physical phenomena occurring with our macroscopic body, like it was the case in Electrodynamics. Next comes our purely theoretical reasoning. For example, although we do not know the body composition, we may sincerely think it is simple, point-like so that three degrees of freedom suffice to describe it. It is the same what we usually think about the electron in Electrodynamics (a point-like object). Now we will try to make ends meet within our simplified "point-like" model.

The main theoretical concern is that, despite Eq. (2·3) is experimentally well established and the constants α and ω have clear physical sense in it, we may think the theory is not fully developed yet – the particle energy possible variations are not related to the oscillator energy gain: $dE_p = \frac{\partial V_{ext}}{\partial t}dt, \quad dE_{osc} = \alpha M_{osc}\dot{\mathbf{r}}_{osc}\cdot\ddot{\mathbf{r}}_p dt$. It looks like while establishing experimentally Newton equation (2·1) the corresponding "radiative losses" were not "noticeable" due to their smallness.

§3. Theory Development I: Better Coupling Equations

Now we theorists may want to intervene in order to "reestablish" the energy conservation law with joining the particle and the wave systems in one total system. Namely, we suppose that Eq. (2·1) is only approximate and needs a small "radiation resistance" force; Eq. (2·3) and its solutions being still perfect as experimentally justified. Thus, our intention and goal is to preserve (2·3) and its solutions as they are and add a "radiation reaction" term into the particle Eq. (2·1) so that the total energy gets conserved. We will proceed from Lagrangians like in CED/QED.

When $\alpha = 0$, Eqs. (2·1) and (2·3) are decoupled and have their own Lagrangians: $L_p^{(0)} = \frac{M_p\dot{\mathbf{r}}_p^2}{2} - V_{ext}\left(\mathbf{r}_p\right)$, $L_{osc}^{(0)} = \frac{M_{osc}\dot{\mathbf{r}}_{osc}^2}{2} - k\frac{\mathbf{r}_{osc}^2}{2}$. In order to derive the searched

"radiation reaction" term, let us try the following "interaction Lagrangian" L_{int}, where $L_{tot} = L_p^{(0)} + L_{osc}^{(0)} + L_{int}$:

$$L_{int} = -\alpha M_{osc}\left(\dot{\mathbf{r}}_p \cdot \dot{\mathbf{r}}_{osc} - \frac{\eta}{2}\dot{\mathbf{r}}_p^2\right). \qquad (3\cdot1)$$

Here the cross term $\propto \dot{\mathbf{r}}_p \cdot \dot{\mathbf{r}}_{osc}$, borrowed from L_{osc} (2·4), gives the desired pumping term $\propto \ddot{\mathbf{r}}_p$ in the wave equation from $\frac{d}{dt}\left(\partial L_{int}/\partial \dot{\mathbf{r}}_{osc}\right)$. But now we think both $\dot{\mathbf{r}}_p$ and $\dot{\mathbf{r}}_{osc}$ must be *unknown* variables in it. Considering $\dot{\mathbf{r}}_p$ unknown variable in (3·1) is our **ansatz** that will make a difference between "insufficiently coupled" Eqs. (2·1), (2·3) and the new ones being derived. We think it will give the searched radiation reaction force.

The quadratic term $\propto \eta\,\dot{\mathbf{r}}_p^2$ in (3·1) simulates here a self-action contribution analogous to the CED/QED electromagnetic mass and is to some extent a stretch in our mechanical problem, but we may always advance it with saying that it does not enter into the oscillator equation, so Lagrangian (3·1) is the most general one satisfying our conditions.

Lagrangian (3·1) is a "self-interaction" Lagrangian similar to $j \cdot A$ in CED/QED where both the current j and its field A are considered unknown and coupled variables. The difference with our mechanical model (2·1), (2·3), (3·1) is mainly in keeping in (3·1) only one oscillator and in assuming finiteness of the "self-energy" contribution $\propto \eta$ because we do not need an infinite η for our modest purposes.

New, "better" coupled mechanical and wave equations are the following:

$$\begin{cases} M_p\ddot{\mathbf{r}}_p = \mathbf{F}_{ext}(\mathbf{r}_p|t) + \alpha M_{osc}\left(\ddot{\mathbf{r}}_{osc} - \eta\,\ddot{\mathbf{r}}_p\right), \\ M_{osc}\ddot{\mathbf{r}}_{osc} + k\,\mathbf{r}_{osc} = \alpha M_{osc}\ddot{\mathbf{r}}_p. \end{cases} \qquad (3\cdot2)$$

At first glance the oscillator equation has not changed and the particle equation has acquired "radiation reaction" terms, as we planned.

3.1. *Energy Conservation in (3·2)*

With Noether theorem or directly from Eqs. (3·2) we obtain the following conservation law (case $\frac{\partial V_{ext}}{\partial t} = 0$):

$$\frac{d}{dt}\left(E_p + E_{osc} + L_{int}\right) = 0. \qquad (3\cdot3)$$

The quantity $E_p + E_{osc} + L_{int}$ is thus conserved. (Here we have a plus sign at L_{int} because L_{int} is of a "kinetic" rather than of "potential" nature.)

The energy conservation law (3·3) does not look as $E_p + E_{osc} = const$. There is an additional term here. However, it is quite similar to the CED conservation law with the "resistance" force $\propto \ddot{\mathbf{v}}_e$ where an extra term is also present in the power balance. Compare the CED conservation law (non relativistic approximation):

$$\frac{d}{dt}\left(E_e + \delta m_e\frac{\dot{\mathbf{r}}_e^2}{2}\right) = -\frac{2e^2}{3c^3}\ddot{\mathbf{r}}_e^2 + \frac{2e^2}{3c^3}\frac{d}{dt}\left(\dot{\mathbf{r}}_e \cdot \ddot{\mathbf{r}}_e\right), \qquad (3\cdot4)$$

with ours:

$$\frac{d}{dt}\left(E_p + \alpha M_{osc}\eta\frac{\dot{\mathbf{r}}_p^2}{2}\right) = -\dot{E}_{osc} + \alpha M_{osc}\frac{d}{dt}\left(\dot{\mathbf{r}}_p \cdot \dot{\mathbf{r}}_{osc}\right). \tag{3·5}$$

For an impulse force the last ("extra") terms with full derivatives do not disappear. But for a quasi-periodical particle motion we may expect, at least on average, the extra term contribution $\left\langle \frac{d}{dt}\left(\dot{\mathbf{r}}_p \cdot \dot{\mathbf{r}}_{osc}\right)\right\rangle$ to vanish in the finite-difference energy balance, like in CED. (The analogy of (3·5) with (3·4) for a quasi-periodical motion is the best for a resonance frequency oscillator with a growing average energy.) Hence, we may hope the energy conservation law has been "reestablished" with advancing (3·2) in the same way as it was done in CED.

The zeroth-order approximation of (3·2) ($\alpha = 0$) corresponds to the "elastic" particle behavior – whatever the external force is, no oscillations are excited. Thus, our system behavior in this approximation is similar to that of QED – the textbooks have plenty of such "elastic" scattering results: the Mott (Rutherford), Bhabha, Klein-Nishina elastic cross sections, etc. In the first perturbative order we obtain "radiative losses" and creation of waves of frequency ω due to the particle acceleration $\ddot{\mathbf{r}}_p^{(0)}(t)$ (a la Bremsstrahlung), etc. In other words, at first (perturbative) glance the new system (3·2) may look acceptable and we may think we have good theoretical principles (recipe $L_{tot} = L_p^{(0)} + L_{osc}^{(0)} + L_{int}$) for constructing interacting theories.

3.2. *Discussion of equation system (3·2)*

Let us see whether we really achieved what we wanted while reestablishing the energy conservation law. The system (3·2) can be cast in the following form:

$$\begin{cases} \tilde{M}_p\ddot{\mathbf{r}}_p = \mathbf{F}_{ext}(\mathbf{r}_p|t) + \alpha M_{osc}\ddot{\mathbf{r}}_{osc}, \\ \tilde{M}_{osc}\ddot{\mathbf{r}}_{osc} + k\,\mathbf{r}_{osc} = \alpha\left(M_{osc}/\tilde{M}_p\right)\mathbf{F}_{ext}(\mathbf{r}_p|t), \end{cases} \tag{3·6}$$

where

$$\tilde{M}_p = M_p\left(1 + \eta\alpha\frac{M_{osc}}{M_p}\right),\ \tilde{M}_{osc} = M_{osc}\left(1 - \alpha^2\frac{M_{osc}}{\tilde{M}_p}\right). \tag{3·7}$$

Here in the mechanical equation we joined two acceleration terms in one and then we inserted the term $\ddot{\mathbf{r}}_p$ from the new mechanical equation into the oscillator one to express it via the external force. According to "particle" Eq. (3·6), the particle inertial properties have changed: with a given external force the particle solution $\mathbf{r}_p(t)$ will manifest quite a different behavior now due to the self-action contribution $\propto \eta$. For example, in case of a low-frequency external force when the "radiation reaction" is really negligible ($\omega_{ext} << \omega$), the equation solution corresponds to a motion of a modified mass $\tilde{M}_p$ in the external field. On the other hand, if we weigh our particle with a spring scale in a static experiment $\left(\mathbf{F}_{ext}\right)_z = M_p g - K\cdot z = 0$, we naturally obtain M_p as a particle mass. Gravity force term, if present in $\mathbf{F}_{ext}$, does not acquire any "mass correction" due to our "coupling" (3·1), so the "bare" mass M_p (if we decide now to call it "bare") and the "correction" $\eta\alpha M_{osc}$ are in principle experimentally distinguishable. Here the situation is quite similar to that in CED

despite it is often erroneously said that m_e and δm_{em} come always in sum and are "indistinguishable".

Our oscillator equation has changed too: the oscillator kinetic term in (3·6) and the coupling constant α have also acquired additional factors. It means, even the oscillator *proper* frequency will be different now (i.e., the frequency before and after the external force acting).

But why has the latter happened? It was not immediately visible in the perturbative treatment of (3·2) mentioned briefly above, but our replacing a known time-dependent function $f(t) = \ddot{\mathbf{r}}_p(t)$ (a solution to (2·1)) with unknown (searched) variable $\ddot{\mathbf{r}}_p$, which is in turn coupled to $\ddot{\mathbf{r}}_{osc}$ in (3·2), *was a strong intervention* into our oscillator equation. If in our development we were proceeding from the numerically equivalent wave equation with the driving force $f(t) = \mathbf{F}_{ext}/M_p$ instead of $\ddot{\mathbf{r}}_p$ in the right-hand side, we would not spoil the wave equation with adding the radiation reaction to the mechanical equation. In other words, the oscillator equation appearance in (3·2), i.e., its similarity to (2·3), is deceptive and misleading. Not having noticed this was our elementary mathematical error and thus we have changed the oscillator equation contrary to our intention. With dividing by $\tilde{M}_{osc}$ it can be written now in the form:

$$\begin{cases} \ddot{\mathbf{r}}_{osc} + \tilde{\omega}^2 \mathbf{r}_{osc} = \tilde{\alpha}\frac{\mathbf{F}_{ext}}{M_p},\ \tilde{\omega} = \omega\left(1 - \alpha^2 M_{osc}/\tilde{M}_p\right)^{-1/2}, \\ \tilde{\alpha} = \alpha\left[\left(1 + \eta\alpha\frac{M_{osc}}{M_p}\right)\left(1 - \alpha^2 M_{osc}/\tilde{M}_p\right)\right]^{-1} \end{cases} \tag{3·8}$$

that shows a change of the proper frequency as well as of the coupling constant α. In other words, our theory development (2·1), (2·3) $\to$ (3·2) via (3·1) has worsened agreement of the new equations with experiments. Now, what is the use of a formal energy conservation law (3·3) if new Eqs. (3·2) describe some "physical systems" quite different from the original ones? The system (3·2) is not acceptable as it is. By the way, if the system (3·2) is solved by the perturbation theory (like in QED), these "increments of constants" are only visible in higher perturbative orders, just like in QED.

It must be clear that such our way of coupling Eqs. (2·1) and (2·3) (our guess (3·1)) is mathematically erroneous since we do not reach our goal with it, but spoil our equations. Replacing a known function with unknown and coupled variable turns out mathematically erroneous in our case. Thus, the self-interaction Lagrangian is wrong at least for the purely mathematical reasons explained above and we must honestly revise our approach.

In CED and QED there was a period of searching for better formulations of theory (H. Lorentz, M. Born, L. Infeld, P. Dirac, R. Feynman, A. Wheeler, F. Bopp, F. Rorhlich to name a few); however, no satisfactory equations were proposed because of lack of right physical ideas. And unfortunately, renormalizations of the fundamental constants happened to lead to good perturbative solutions in some rare, but important cases; that is why constructing *renormalizable* theories has gradually become the mainstream activity. Nowadays the fact of coefficient modifications due to "interaction" similar to (3·1) in QFT is used not as evidence of the coupling term

being wrong, but as a "proof" of the original constants (and particles) being non physical, "bare" ones.[1)] The latter can be called a "theoretical discovery of bare particles and their physics".

In order to make our construction (3·6) work, we too are going first to follow the renormalization prescription. Fortunately, in (3·6) it can be done exactly.

§4. Development II: Renormalizations in equations (3·6)

Speaking specifically of our particle, its mass renormalization means calling the whole combination $\tilde{M}_p$ "the physical mass" and using for it the old numerical value M_p from (2·1). This is equivalent to discarding the "correction" $\propto \eta$ to M_p in (3·7). After that we restore at least the right inertial properties of our solution $\mathbf{r}_p(t)$. We do not know yet if the resulting equation becomes good for the radiation reaction description, but hope for it.

As well, we see that the particle mass renormalization is not sufficient to "repair" system (3·6): the oscillator mass (or proper frequency) and the coupling constant (3·8) are still different from those in (2·3). In order to restore the right constants, it is sufficient now to renormalize the oscillator mass $\tilde{M}_{osc}$ in (3·7) with discarding the term proportional to α^2. Thus, with only two "independent" renormalizations we can advance an exactly renormalized equation system:

$$\left\{ \begin{array}{l} M_p \ddot{\mathbf{r}}_p = \mathbf{F}_{ext}(\mathbf{r}_p|t) + \alpha M_{osc} \ddot{\mathbf{r}}_{osc}, \\ M_{osc} \ddot{\mathbf{r}}_{osc} + k\, \mathbf{r}_{osc} = \alpha M_{osc} \frac{\mathbf{F}_{ext}(\mathbf{r}_p|t)}{M_p}. \end{array} \right. \tag{4·1}$$

This system contains all physical constants practically in the same way as in the original phenomenological equations (2·1) and (2·3). As well, after renormalizations a net "radiation reaction" force was left in the mechanical equation.

In terms of interaction Lagrangian, this exact renormalization is equivalent to and can be implemented as adding the following "counter-term" L_{CT} to our "bare" (or trial) Lagrangian $L_{Bare} = L_p^{(0)} + L_{osc}^{(0)} + L_{int}$:

$$L_{Phys} = L_{Bare} + L_{CT} = L_{Bare} - \alpha\eta \frac{M_{osc} \dot{\mathbf{r}}_p^2}{2} + \alpha^2 \left(\frac{M_{osc}}{M_p} \right)^2 \frac{M_p \dot{\mathbf{r}}_{osc}^2}{2} \tag{4·2}$$

It means subtracting from (3·1) the potential self-action contribution proportional to η (it has never been useful), leaving there the cross term $\propto \dot{\mathbf{r}}_p \cdot \dot{\mathbf{r}}_{osc}$ (partially needed for coupling equations (2·1) and (2·3), and adding a quadratic in α kinetic term to the oscillator kinetic energy to cancel the oscillator mass modification arising due to that cross term. Later we will see that $L_{int} + L_{CT}$ in (4·2) is not only a better interaction Lagrangian to be treated by same the perturbation theory. In fact, some part of $L_{int} + L_{CT}$ can and should be taken into account exactly into the new zeroth-order approximations that will give different (improved) perturbation expansions with improved description of physics.

4.1. *Discussion of equation system (4·1)*

Now, let us analyze our exactly renormalized equation system (4·1) and its physics. First of all, the oscillator equation in it is rather "decoupled" from the particle one – it is influenced directly with the external force (the total decoupling occurs in case of a uniform external force $\mathbf{F}_{ext}(t)$). It should not be a big surprise though since such a feature was correctly implied already in (2·2) with (2·1). So Eq. (4·1) for $\mathbf{r}_{osc}$ has fortunately the right solutions and we must keep it.

But let us look at the particle equation: $\mathbf{r}_p$ is now influenced with the oscillator motion in a "one-way" way! The external force of a limited duration T excites the oscillator that oscillates freely afterwards, but the particle gets these free oscillations as an external known force now: $\ddot{\mathbf{r}}_p \propto \ddot{\mathbf{r}}_{osc}(t)$. Did we expect this "feedback" from the oscillator in the beginning of our program on reestablishing the energy conservation law? Didn't we expect decoupling equations in absence of external forces? Meanwhile the force $\propto \ddot{\mathbf{r}}_{osc}$ is now always present in the particle equation. What should we think of our system (4·1)? Wrong again? Here a more fine comparison with experimental data can give us an ultimate answer. And let us suppose that our experimentalists discover, with sophisticated optical and acoustical measurements, that indeed, after force acting our probe body actually vibrates *as a whole* (rather than changes its shape) in a qualitative agreement with the "vibrating" solution $\mathbf{r}_p$ from (4·1), so the mechanical part of (4·1) is also right. We do not need to repair any equation anymore, fortunately. We are certainly lucky and now it is our understanding that needs a repair.

Comparing the original Eqs. (2·1) and (2·3) with the correctly coupled ones (4·1), we conclude that in order to prevent the oscillator equation coefficients from modifications due to introducing the "radiation reaction" term, we should have simply expressed the oscillator right-hand side via the external force (see **Remark**).

Thus, the oscillating "external force" $\propto \ddot{\mathbf{r}}_{osc}$ in the particle Eq. (4·1) is the right "radiation reaction" term we have been looking for. We wanted a better coupling, a feedback from the oscillator in the particle equation and we got it. It lasts longer than foreseen, but now the particle acceleration $\ddot{\mathbf{r}}_p$ does not influence the oscillator when the force ceased acting ($\mathbf{F}_{ext} = 0$). How can the latter be? It can be so only if our particle belongs to the oscillator and conversely. The particle oscillations $\ddot{\mathbf{r}}_p \propto \ddot{\mathbf{r}}_{osc}(t)$ do not serve as a "pumping term" for the oscillator if the particle is just an oscillator piece. This is the right understanding of equation coupling contained in (4·1). To see it better, let us introduce another dynamical variable (we join kinetic terms in (4·1)):

$$\mathbf{R} = \mathbf{r}_p - \alpha \frac{M_{osc}}{M_p} \mathbf{r}_{osc}. \tag{4·3}$$

Then we obtain the equations:

$$\left\{ \begin{array}{l} M_p \ddot{\mathbf{R}} = \mathbf{F}_{ext}(\mathbf{R} + \alpha \frac{M_{osc}}{M_p} \mathbf{r}_{osc} | t), \\ M_{osc} \ddot{\mathbf{r}}_{osc} + k\, \mathbf{r}_{osc} = \alpha \frac{M_{osc}}{M_p} \mathbf{F}_{ext}(\mathbf{R} + \alpha \frac{M_{osc}}{M_p} \mathbf{r}_{osc} | t). \end{array} \right. \tag{4·4}$$

If the external force is uniform ($\mathbf{F}_{ext}(t)$), the variable $\mathbf{R}$ does not have oscilla-

tions even though the force is on. Also, after the force ceased acting, the equation for $\mathbf{R}$ describes a free motion. It looks as an equation for the center of mass (CM) of a coupled system and the oscillator equation can be understood now as an equation for the relative motion in a coupled system. Hitting our particle excites the relative/internal motion in the system and transfers some kinetic energy to the system as a whole. This is what the correct Eqs. (4·4) say. It may well happen if the body is a *coupled* (compound) system and the external force only acts on *one of constituents* of our body. A feasible mechanical model for such a system is already given in Fig. 1, but now we have inferred it exclusively from Eqs. (4·4) physical analysis.

Thus, we figured out the right physics of coupling from the correct equations. It is very different from the "bare particle physics". The energy conservation law for such a system reads: the external force work $-\varDelta V_{ext}$ done on displacing the constituent particle from $\mathbf{r}_p(t_1)$ to $\mathbf{r}_p(t_2)$ is spent on changing the center of mass kinetic energy and on changing the relative motion (internal) energy of the compound system, both works being additive:

$$\begin{cases} -\varDelta V_{ext} = \varDelta\left(\frac{M_p\dot{\mathbf{R}}^2}{2}\right) + \varDelta E_{osc}, \\ E = \frac{M_p\dot{\mathbf{R}}^2}{2} + V_{ext}\left(\mathbf{R} + \alpha\frac{M_{osc}}{M_p}\mathbf{r}_{osc}\right) + \frac{M_{osc}\dot{\mathbf{r}}_{osc}^2}{2} + k\frac{\mathbf{r}_{osc}^2}{2}. \end{cases} \tag{4·5}$$

Also it is clear that M_p is in fact the *total mass* of the system: $M_p = M_{tot}$, and M_{osc} is the reduced mass μ involved in the relative motion equation. Indeed, if we take a couple of particles (constituents) with m_1 and m_2 coupled with a spring k and introduce the corresponding variables $\mathbf{R} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/(m_1 + m_2)$ and $\mathbf{r}_{osc} = \mathbf{r}_1 - \mathbf{r}_2$, we will obtain equations like (4·4) with $M_p = M_{tot} = m_1 + m_2$, $M_{osc} = \mu = \frac{m_1 m_2}{m_1+m_2}$, and $\alpha = (m_1 + m_2)/m_1$. The true oscillator coordinate $\mathbf{r}_{osc}$ describes the relative/internal motion, the oscillating part of $\mathbf{r}_1 = \mathbf{r}_p$ and thus the sound amplitude being just proportional to it: $\mathbf{r}_p = \varepsilon\mathbf{r}_{osc} + \mathbf{R}$, $\varepsilon = m_2/(m_1 + m_2)$. Lagrangian (4·2) in these variables is the following:

$$L_{Phys} = \frac{M_p\dot{\mathbf{R}}^2}{2} - V_{ext}\left(\mathbf{R} + \varepsilon\mathbf{r}_{osc}\right) + \frac{M_{osc}\dot{\mathbf{r}}_{osc}^2}{2} - k\frac{\mathbf{r}_{osc}^2}{2}. \tag{4·6}$$

Equation coupling occurs here via the argument of the external force potential $V_{ext}(\mathbf{r}_p)$, not via a product like in (3·1). It is a good news for QED where couplings are implemented as field products which lead to "colliding infinities" that get into the equation coefficients. QED equations made physically analogous to (4·4) will hopefully describe the occupation number evolutions without renormalizations. Indeed, the QED equations like a particle in a known external potential and a field due to a known source have often reasonable physical solutions.

By the way, equation system (4·4) has an advantage over the others because it is in fact a more general one and is the only applicable formulation in situations when one does not know the studied body "composition". Indeed, in a more realistic case when the body cannot be disassembled into simple mechanical pieces like m_1, m_2, and a spring k, it is still possible to study experimentally the center of mass motion and the normal modes of the compound system in question, so an equation system a la (4·4) is the right phenomenological framework for that.

4.2. *Perturbation theory for equations (4·4)*

The external force argument $\mathbf{r}_p = \mathbf{R} + \varepsilon\,\mathbf{r}_{osc}$ is different from $\mathbf{R}$, so the equations in (4·4) are coupled in general case. It may be convenient to expand the force "around" $\mathbf{R}$ if the force gradient is relatively weak. This gradient is another small parameter *different from* α; the latter is already involved in the zeroth-order approximations $\mathbf{r}_{osc}^{(0)}$ and $\mathbf{r}_p^{(0)} = \mathbf{R}^{(0)} + \varepsilon\mathbf{r}_{osc}^{(0)}$. No conceptual and/or mathematical difficulties can be expected on this way since the exact equations (4·4) are physical, have physical solutions, and the zeroth-order equations with $\mathbf{F}_{ext}\,(\mathbf{R})$ in each of them capture already well the exact solution properties. In particular, both in Classical and Quantum Mechanical treatments *the soft oscillator modes are automatically excited in this approximation* and an inclusive (average) picture becomes unavoidable and natural. The latter result is achieved in QED only with a heavy summation of soft perturbative contributions to all orders because nothing from the $L_{int} + L_{CT}$ is included into the zeroth-order approximation (α still being the expansion parameter there). Note, equations (4·4) in the zeroth order have the same numerical solutions as our original equations (2·1) and (2·3). Thus, transition from the approximate equations (2·1), (2·3) to the exact ones (4·4) may be achieved with "enlarging" the external force argument if these equations are understood correctly.

§5. General Discussion

Passing through our "theory development stages" I and II, we were in fact discovering complexity of our material body. We sincerely thought the body was point-like (we applied an equation for one point!) and may well be decoupled from the "wave" system, but exactly renormalized equations supported by experiments taught us it is not. Point-like and "deterministic" is the center of mass instead. The external force acts in fact on one of the constituents of an interacting system and the latter has "internal" degrees of freedom observed as oscillations. Correct equations (4·4) are quite comprehensible and familiar to us. They are quasi-particle equations of a compound system describing the global (CM) and the relative (internal) collective motions. If the external force is uniform, they even coincide in form with the original ones (2·1) and (2·3) where $\ddot{\mathbf{r}}_p(t)$ in (2·3) is replaced with $\mathbf{F}_{ext}(t)/M_p$. In this particular case the energy conservation law *already holds* and it is so just because of different physical meaning of (separated) variables $\mathbf{R}$ and $\mathbf{r}_{osc}$! We should not couple these equations at all and we could have even deduced Eqs. (4·4) directly from (2·1) and (2·3) if we had initially admitted the right physical idea about our body being compound. This is what can be called a reformulation approach. Indeed, our renormalized equations, especially in form (4·4), are obviously equivalent to a theory *formulated from a different physical concept*– an idea whose necessity was so persistently promoted by P. Dirac.[2)] In our toy model these right physical ideas are a compound character of the probe body, belonging the constituent particle-1 (point of the force application) to the oscillator, and oscillator being an "internal degree of freedom" of this compound system. Such ideas would prevent us from advancing wrong equations with subsequent renormalizations of coefficients in them.

We treated our body (Figs. 1, 2) as simple, point-like, for two main reasons: an experimental and a human one. We humans tend to deal with as simple things as possible. And experimentally, even if we monitor the true particle-1 (oscillating) coordinate $\mathbf{r}_p(t)$, our experimental results may give us the center of mass (smooth) coordinate due to certain averaging. The permanent coupling of $\mathbf{r}_p$ and $\mathbf{r}_{osc}$ in the relationship $\mathbf{r}_p = \mathbf{R} + \varepsilon\,\mathbf{r}_{osc}$ may be lost, for example, due to time averaging: $\langle\mathbf{r}_p\rangle \approx \langle\mathbf{R}\rangle$. Thus, we observe mostly a quasi-particle coordinate $\langle\mathbf{R}\rangle$ and we think it is a particle, microscopic one $\mathbf{r}_p$. So one of the roots of our physical error in the theory development was in our misunderstanding the phenomenological equation (2·1) established factually for $\langle\mathbf{R}\rangle$. We took average, inclusive, "macroscopic" thing $\langle\mathbf{R}\rangle$ for a "microscopic" one $\mathbf{r}_p(t)$. Average (inclusive) character of some experimental notions is important for their observability/certainty/determinism, but we idealize these notions and forget that they are built in reality due to summation (inclusive observation) and do not exist independently of them. In addition to this, the same permanent coupling existing in the wave equations is written historically in a way not revealing the right physics ($\propto \ddot{\mathbf{r}}_p$ rather than $\propto \mathbf{F}_{ext}$); thus, it is not considered as permanent although it is such. Then attempts to couple coupled already things fail and we need "renormalizations" and other weird "inventions" on the go to get out of our theoretical impasse.

We did not take the damping explicitly, but we meant it: the sound observations leading to Eq. (2·3) are only possible due to oscillator's gradually transmitting its energy to the air or directly to a measuring device. Even after damping out oscillations, our steady particle-1 remains permanently coupled within the oscillator and ready for new adventures. In Quantum Mechanics such a charge permanently coupled within the electromagnetic field oscillators (called an "electronium") is described with an elastic and inelastic state-dependent form-factors[3)] briefly outlined in Chapter 4. In 1948 T. Welton even proposed something analogous to (4·1), (4·6): his electron was permanently influenced with an "external" force of electromagnetic field oscillators and that led to the main part of the Lamb shift.[4)] However, his estimations were considered qualitative, probably because such a "one-way" influence was hard to imagine (oscillators exist "everywhere in space" and influence the electron, but not vice versa). Had he figured out that the electron belonged to the field oscillators and that the latter described the *relative, collective motions* in a compound system (quasi-particles), the QED development might have taken another route.

Above we arrived at the right equation system after fulfilling an exact renormalization of two masses. Without profound analysis it may give an impression that renormalization is a good way of doing physics as it works. But let us not fool ourselves. Renormalizations may sometimes work because the correct equations (perturbative solutions in QFT) may be so simple that they can be guessed right from the obviously wrong ones. We constructed coupled equations (3·2) from the original ones (2·1), (2·3), where the original ones worked nearly fine: the fundamental constants are defined precisely from them. To couple better (2·1) and (2·3) we first introduced wrong kinetic terms that couple equations indeed, but such a trial coupling "modified masses" (equation coefficients), and then we decided to discard these modifications. Both our actions (2·1), (2·3)→(3·2) and (3·6)→(4·1) nearly can-

celed each other – the unexpected and unnecessary mass "corrections" were removed by hand (4·2). There was only a little chance that the remainder of these "development stages" would be good and Eqs. (2·1), (2·3) would become coupled correctly in the end. CED with its runaway solutions and non renormalizable QFT are a bright example of a failure of such an "approach".

Since it is we who changed the coefficients twice, it is useless to study relationships between "bare" constants and "physical" ones: there were no bare ones in (2·1) and (2·3), but quasiparticle parameters M_{tot}, μ, ω, etc. The notion of a "bare" constant with a "wild value" was invented by people when they tried to keep bad (wild) corrections as *legitimate* ones and at the same time to not contradict the experimental measurements; thus the original constant was made "guilty". In reality it is the coefficient corrections δm, δe due to the corresponding theoretical trial construction who are bad, not the original constants like ours M_p, M_{osc}, ω, and α. Hence, a "bare particle physics" is not a real physics at all, but a weird picture (interpretation) imposed when people postulate wrong theoretical constructions as right and force them to describe the reality. Only a strong belief in correctness and uniqueness of the wrongly coupled equations and an *accidental* "success" of renormalizations make some accept this "bare particle physics". Indeed, the very first effect of coupling to something in quantum mechanics is quantum mechanical smearing and energy level formation rather than "classical vacuum polarization" around a "steady bare charge". As well, the absence of soft radiation in the first Born approximation in QED (i.e., obtaining elastic processes) is a crying sign of a bad initial approximation apparently caused with a too superficial understanding of how the correct coupling must in reality be done (see (4·4) and Subsection 4.2).

§6. Conclusions

We have seen that experimentally established equations (2·1) and (2·3) could be coupled in a wrong way like in our simple mechanical case of "ringing a bell" due to our physical and mathematical errors. First we advanced a wrong interaction Lagrangian (3·1) by analogy with (2·4) that looked natural and innocent; then we modified the obviously wrongly coupled equations with renormalizing masses in their kinetic terms. In our toy model the renormalizations could be fulfilled exactly in equations or in the total Lagrangian. Renormalized equations turned out to be the right ones, however they revealed a different and surprising physics – that of permanently coupled constituents. We figured out that our phenomenological "mechanical" and "wave" equations (2·1) and (2·3) corresponded factually to the exact equations (4·4) written in terms of separated variables (center of mass $\mathbf{R}$ and relative motion variables), that is why they should have been coupled differently. Fortunately the right microscopic equations were so simple that the renormalizations were practically the only "repair" to obtain them. Finally we understood that the observed masses and frequencies corresponded to quasi-particles (or normal modes) of our compound (interacting) system, and quasi-particles are a familiar and a natural language for interacting systems. In this respect our toy model is quite instructive. It means our original phenomenological equations could have been coupled in a differ-

ent, correct way directly if we had initially had the right physical ideas about the observed phenomenon. The mechanical model from Fig. 1 proves it. This whole story is quite similar to QED except for in QED the right understanding of coupling has not been reached yet. Indeed, originally in QED the interaction Lagrangian was meant to simply change the occupation numbers of particles, but it gives corrections to the equation coefficients too. The latter drawback is repaired with the coefficient renormalizations which is generally equivalent to a theory reformulation. Hence, the accidental "success of renormalizations" in QFT should be understood as a strong invitation to revise the original (wrong) physics including wrong coupling for the sake of having an initially better physical formulation which is hopefully feasible. A "quasi-particle formulation" of QFT is not so difficult to accept and we should not be embarrassed to construct it. It will help eliminate wrong and harmful notions from physics as well as reformulate some non renormalizable theories.

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